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Radiative corrections for pion and kaon production at e^+e^- colliders of energies below 2 GeV

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Abstract: Processes of electron–positron annihilation into charged pions and kaons are considered. Radiative corrections are taken into account exactly in the first order and within the leading logarithmic approximation in higher orders. A combined approach for accounting exact calculations and electron structure functions is used. An accuracy of the calculation can be estimated about 0.2%.

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Contents

1 Introduction

The processes in collisions of e^+e^- beams at moderate high energies with detection of the final particles moving in centre–of–mass system (c.m.s.) at large angles are the subject of close attention at meson factories, such as VEPP–2M (Novosibirsk) [1], DAΦNE (Frascati) [2], BEPC/BES (Beijing) [3]. The processes of pure quantum electrodynamics (QED) nature provide an important background for studies of subtle mesons properties. Besides, they may be used for a calibration and monitoring. Because of large cross-sections of the lowest order processes, radiative corrections (RC) to them are to be included in the consideration.

In our previous paper [4] we had considered the QED processes $e^+e^- \to e^+e^-(\gamma)$, $\mu^+\mu^-(\gamma)$, $\gamma\gamma(\gamma)$. There we developed an approach for precise accounting of radiative corrections to differential distributions. The part of RC, describing the emission of hard additional photons was presented in the form convenient for imposing experimental conditions of the final particle detection. The contribution of higher orders of perturbation theory was considered in the leading logarithmical approximation. That was done by means of the structure function approach, writing a cross-section in the Drell-Yan form.

In this paper we apply the same approach to processes $e^+e^- \to \pi^+\pi^-(\gamma)$, $K_LK_S(\gamma)$, $K^+K^-(\gamma)$, considering the pseudoscalar mesons as point–like objects. The effects of strong interaction of hadrons in the final state are parametrized by introducing form factors which are to be measured in an experiment. We assume as usually that vacuum polarization corrections (by hadrons and leptons) are also included in the form factors.

The virtual and soft real photon emission corrections are calculated in the $\mathcal{O}(\alpha)$ order exactly. This permits us to keep explicitly the leading (containing *large* logarithm $L = \ln(s/m_e^2)$, $s = 4\varepsilon^2$ is the squared total energy in c.m.s.) and next–to–leading terms. The latters are regarded further as $\mathcal{K}$–factor terms in the Drell–Yan representation for a cross-section.

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Considering hard photon emission we extract contributions, containing large logarithm L, we distinguish the collinear kinematics of hard photon emission and the semi–collinear ones, which do not give rise to L. An auxiliary parameter, a small polar angle $\theta_0 \ll 1$ with respect to the direction of the initial beams, is introduced for this purpose. The terms containing photon softness parameter $\Delta = \Delta\varepsilon/\varepsilon$ ($\Delta\varepsilon$ is the maximal energy of a soft (in c.m.s.) undetected photon) and the ones containing θ_0 from the contribution of the collinear region are regarded as compensating terms. When they being summed with the contribution of the semi–collinear region (with the relevant restrictions imposed) provide their finiteness in the limit $\Delta, \theta_0 \to 0$.

This paper is organized as follows. In the second Section we consider the process of charged pion production. The explicit formulæ (we keep pion mass exactly) for the lowest order virtual and soft real photon emission are presented. The charge–even and charge–odd contributions are given separately. The latter quantity permits us to obtain the charge asymmetry which can be measured. We put also the explicit formula for the differential cross-section of hard photon emission. In Sect. 3 a similar consideration is given for the case of neutral and charged kaon production near the threshold. In Conclusions we discuss the formulæ obtained and their precision. The results are illustrated numerically in Figures. In the Appendix we discuss the structure of form factors and give the explicit expressions for the vacuum polarization operator.

2 Pion pair production

In the Born approximation the differential cross-section of the process

$$e^+(p_+) \ + \ e^-(p_-) \ \to \ \pi^+(q_+) \ + \ \pi^-(q_-) \qquad (2.1)$$

has the form

$$\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}(s) = \frac{\alpha^2\beta^3}{8s}\sin^2\theta\,|F_\pi(s)|^2, \qquad (2.2)$$

$$\beta = \sqrt{1 - m_\pi^2/\varepsilon^2}, \qquad s = (p_+ + p_-)^2 = 4\varepsilon^2, \qquad \theta = \widehat{\boldsymbol{p}_-\boldsymbol{q}_-}\,.$$

The pion form factor $F_\pi(s)$ takes into account vertex virtual corrections due to strong interactions and vacuum polarization by leptons and hadrons (including vector–meson resonances) [5] (see Appendix). We would like to underline that in our approach QED corrections are not included into $F_\pi(s)$. One has to take the form factor from an experiment after an extraction of QED radiative corrections.

Calculating QED radiative corrections we will consider pion as a point–like particle. We distinguish form–factor–type one–loop Feynman diagrams and the box–type ones. The QED form factors are taken as $F^{QED}_{e,\pi}(s) = 1 + F^{(1)}_{e,\pi} + \mathcal{O}(\alpha^2)$. Using the known first order contributions to the electron and pion QED form factors

$$\mathrm{Re}\ F_e^{(1)}(s) \ = \ \frac{\alpha}{\pi}\biggl[\biggl(\ln\frac{m_e}{\lambda} - 1\biggr)(1 - L) - \frac{1}{4}L^2 + \frac{\pi^2}{3} - \frac{1}{4}L\biggr], \qquad L = \ln\frac{s}{m_e^2}\,, \qquad (2.3)$$

$$
\begin{aligned}
\mathrm{Re}\, F_\pi^{(1)}(s) \;&=\; 1+\frac{\alpha}{4\pi}\Big\{4\left(\ln\frac{m_\pi}{\lambda}-1\right)\left(1-\frac{1+\beta^2}{2\beta}\, l_\beta\right) \\
&+\frac{1+\beta^2}{2\beta}\Big[-2\ln\left(\frac{4}{1-\beta^2}\right)\, l_\beta+\ln^2\frac{1-\beta}{2}-\ln^2\frac{1+\beta}{2} \\
&-2l_\beta\ln\beta-2\mathrm{Li}_2\left(-\frac{1-\beta}{2\beta}\right)+2\mathrm{Li}_2\left(\frac{1-\beta}{2\beta}\right)\Big]\Big\}, \qquad l_\beta=\ln\frac{1+\beta}{1-\beta},
\end{aligned} \tag{2.4}
$$

and the contribution due to soft photon emission [6, 7], we obtain for the charge–even part of the differential cross-section the following formula:

$$
\frac{\mathrm{d}\sigma_{\mathrm{even}}^{B+S+V}}{\mathrm{d}\Omega}=\frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\Big\{1+\frac{2\alpha}{\pi}[A+B]\Big\}, \tag{2.5}
$$

with:

$$
\begin{aligned}
A \;&=\; (L-1)\ln\frac{\Delta\varepsilon}{\varepsilon}+\frac{3}{4}(L-1)+a, \qquad a=\frac{\pi^2}{6}-\frac{1}{4}, \\
B \;&=\; \left(\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\right)\ln\frac{\Delta\varepsilon}{\varepsilon}+b(s), \\
b(s) \;&=\; -1+\frac{1-\beta}{2\beta}\rho+\frac{1}{\beta}\ln\frac{1+\beta}{2}+\frac{1+\beta^2}{2\beta}\Big[-\mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right) \\
&+\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)-\frac{\pi^2}{12}+\rho\ln\frac{1+\beta}{2}-2\rho\ln\beta+\frac{3}{2}\ln^2\frac{1+\beta}{2} \\
&-\frac{1}{2}\ln^2\beta-3\ln\beta\ln\frac{1+\beta}{2}+\rho+2\ln\frac{1+\beta}{2}\Big], \\
\rho \;&=\; \ln\frac{s}{m_\pi^2}\,, \qquad \mathrm{Li}_2(x) \;=\; -\int_0^x\frac{\mathrm{d}t}{t}\ln(1-t).
\end{aligned}
$$

Considering the box–type diagrams, one has to note, that the corresponding integral over the loop momentum is convergent. So, the contribution of large virtual momenta is negligible, and we do not need to consider the Wilson operator expansion there. Box–type diagrams and the interference of soft photon emission from electrons and pions give rise for the charge–odd contribution

$$
\frac{\mathrm{d}\sigma_{\mathrm{odd}}^{S+V}}{\mathrm{d}\Omega} \;=\; \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}\,\frac{2\alpha}{\pi}\Big\{2\ln\frac{\Delta\varepsilon}{\varepsilon}\ln\frac{1-\beta c}{1+\beta c}+k(c,s)\Big\}, \tag{2.6}
$$

with:

$$
\begin{aligned}
k(c,s) \;&=\; \frac{1}{2}l_-^2-\mathrm{Li}_2\left(\frac{1-2\beta c+\beta^2}{2(1-\beta c)}\right)+\mathrm{Li}_2\left(\frac{\beta^2(1-c^2)}{1-2\beta c+\beta^2}\right) \\
&-\int_0^{1-\beta^2}\frac{\mathrm{d}x}{x}f(x)\left(1-\frac{x(1-2\beta c+\beta^2)}{(1-\beta c)^2}\right)^{-\frac{1}{2}} \\
&+\frac{1}{2\beta^2(1-c^2)}\Big\{\Big[\frac{1}{2}l_-^2-(\rho+l_-)L_-+\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right)\Big](1-\beta^2)+
\end{aligned}
$$

$$
\begin{aligned}
& + (1-\beta c)\Big[-l_-^2 - 2\mathrm{Li}_2\left(\frac{1-\beta^2}{2(1-\beta c)}\right) + 2(\rho + l_-)L_- - \frac{(1-\beta)^2}{2\beta}\left(\frac{1}{2}\rho^2 + \frac{\pi^2}{6}\right) \\
& + \frac{1+\beta^2}{\beta}\Big(\rho \ln\frac{2}{1+\beta} - \mathrm{Li}_2\left(-\frac{1-\beta}{1+\beta}\right) + 2\mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\Big)\Big]\Big\} - (c \to -c), \\
f(x) &= \Big(\frac{1}{\sqrt{1-x}} - 1\Big)\ln\frac{\sqrt{x}}{2} - \frac{1}{\sqrt{1-x}}\ln\frac{1+\sqrt{1-x}}{2}, \\
l_- &= \ln\frac{1-\beta c}{2}, \qquad L_- = \ln\Big(1 - \frac{1-\beta^2}{2(1-\beta c)}\Big).
\end{aligned}
$$

The charge asymmetry in a quasi–elastic case has the form

$$
\eta = \frac{\mathrm{d}\sigma(c) - \mathrm{d}\sigma(-c)}{\mathrm{d}\sigma(c) + \mathrm{d}\sigma(-c)} = \frac{\mathrm{d}\sigma_{\mathrm{odd}}^{S+V}}{\mathrm{d}\sigma_0}\,. \tag{2.7}
$$

In the ultra–relativistic case ($\beta \to 1$) we obtain

$$
\begin{aligned}
(\eta)_{\mathrm{asympt}} &= \frac{2\alpha}{\pi}\Big[4\ln(\mathrm{tg}\frac{\theta}{2})\,\ln\frac{\Delta\varepsilon}{\varepsilon} + \Big(2 - \frac{1}{\cos^2\frac{\theta}{2}}\Big)\ln^2(\sin\frac{\theta}{2}) \qquad (2.8) \\
& - \Big(2 - \frac{1}{\sin^2\frac{\theta}{2}}\Big)\ln^2(\cos\frac{\theta}{2}) + \mathrm{Li}_2(\cos^2\frac{\theta}{2}) - \mathrm{Li}_2(\sin^2\frac{\theta}{2})\Big], \quad \varepsilon \gg m.
\end{aligned}
$$

This expression coincides with the result of Brown and Mikaelian [8].

The matrix element of the process accompanied by hard photon emission

$$
e^-(p_-) + e^+(p_+) \longrightarrow \pi^-(q_-) + \pi^+(q_+) + \gamma(k) \tag{2.9}
$$

can be presented in the following form:

$$
\begin{aligned}
M^{e^+e^-\to\pi^+\pi^-\gamma} &= -i(4\pi\alpha)^{\frac{3}{2}}\Big\{\bar{v}\Big[\gamma_\nu\left(\frac{p_+e}{\chi_+} - \frac{p_-e}{\chi_-}\right) + \frac{\gamma_\nu\hat{k}\hat{e}}{2\chi_-} - \frac{\hat{e}\hat{k}\gamma_\nu}{2\chi_+}\Big] \times \qquad (2.10) \\
& \times u(q_- - q_+)^\nu \frac{F_\pi(s_1)}{s_1} + \bar{v}\gamma_\rho u \frac{F_\pi(s)}{s}T^\pi_{\rho\sigma}e^\sigma(k)\Big\},
\end{aligned}
$$

$$
p_- + p_+ = q_- + q_+ + k, \quad s = (p_- + p_+)^2, \quad s_1 = (q_- + q_+)^2, \quad \chi_\pm = p_\pm k.
$$

The tensor $T^\pi_{\rho\sigma}$ describes the transition of a *heavy* photon into the system of two real pions and a real photon:

$$
\gamma^*(q) \to \pi^+(q_+) + \pi^-(q_-) + \gamma(k), \quad q_+^2 = q_-^2 = m_\pi^2, \quad q^2 = s, \quad k^2 = 0. \tag{2.11}
$$

Regarding CPT and gauge invariance the tensor can be written in the general form

$$
T^\pi_{\rho\sigma} = a_1 L^{(1)}_{\rho\sigma} + a_2 L^{(1)}_{\rho\sigma} + a_3 L^{(1)}_{\rho\sigma} + k_\sigma O_\rho, \qquad q^\rho T^\pi_{\rho\sigma} = 0, \quad k^\sigma T^\pi_{\rho\sigma} = 0. \tag{2.12}
$$

The last term ($\sim k_\sigma$) is irrelevant here. Tensors $L^{(i)}$ read

$$
\begin{aligned}
L^{(1)}_{\rho\sigma} &= qk\; g_{\rho\sigma} - k_\rho q_\sigma, \qquad L^{(2)}_{\rho\sigma} = qk\; Q_\rho Q_\sigma - kQ\;(q_\sigma Q_\rho + Q_\sigma k_\rho) + (kQ)^2 g_{\rho\sigma}, \\
L^{(3)}_{\rho\sigma} &= kQ\;(q^2 g_{\rho\sigma} - q_\rho q_\sigma) + Q_\sigma(qk\; q_\rho - q^2 k_\rho), \qquad Q = \frac{1}{2}(q_+ - q_-). \qquad (2.13)
\end{aligned}
$$

For the case of charge pion production, which is considered below, we used the approximation of point–like pions, where

$$T^{\pi}_{\rho\sigma} \to T^{(0)}_{\rho\sigma} = \frac{1}{2\chi'_+}(q_- - q_+ - k)_\rho(-2q_+ - k)_\sigma + \frac{1}{2\chi'_-}(q_- - q_+ + k)_\rho(2q_- + k)_\sigma - 2g_{\rho\sigma}\,, \tag{2.14}$$

$$\chi'_\pm = q_\pm k, \qquad a_1^{(0)} = -\frac{2}{\chi'_- + \chi'_+}\left(\frac{\chi'_-}{\chi'_+} + \frac{\chi'_+}{\chi'_-}\right), \qquad a_2^{(0)} = \frac{16}{\chi'_-\chi'_+}, \qquad a_3^{(0)} = 0\,.$$

In reality some vector–meson resonance intermediate states give rise of contributions to the tensor.

The differential cross-section of the process with hard photon emission reads

$$\mathrm{d}\sigma^{e^+e^-\to\pi^+\pi^-\gamma} = \frac{\alpha^3}{2\pi^2 s^2}(R_1 + R_2 + R_3)\mathrm{d}\Gamma, \tag{2.15}$$

with:

$$\begin{aligned}
R_1 &= \frac{s}{s_1^2}|F_\pi(s_1)|^2(p_+Qp_-Q)\Big[\frac{p_+p_-}{\chi_+\chi_-} - \frac{2}{\chi_-} - \frac{m_e^2}{\chi_-^2} \\
&\quad + \frac{\chi_+}{p_+p_-}\left(\frac{1}{\chi_-} + \frac{m_e^2}{\chi_-^2}\right) + (p_+ \leftrightarrow p_-)\Big], \\
R_2 &= \Big\{\frac{1}{s}|F_\pi(s)|^2\Big[\frac{q_+q_-}{\chi'_+\chi'_-} - \frac{m_\pi^2}{(\chi'_+)^2} + (q_+ \leftrightarrow q_-)\Big] \\
&\quad + \frac{2}{s_1}\mathrm{Re}\,(F_\pi(s)F_\pi^*(s_1))\left(\frac{p_+}{\chi_+} - \frac{p_-}{\chi_-}\right)\left(\frac{q_+}{\chi'_+} - \frac{q_-}{\chi'_-}\right)\Big\}(p_+Qp_-Q), \\
R_3 &= \frac{s}{s_1^2}|F_\pi(s_1)|^2\Big[\frac{(p_+Qk_-Q)}{\chi_-} + \frac{(p_-Qk_+Q)}{\chi_+} + \frac{2Qk}{\chi_+\chi_-}(p_+Qp_-k)\Big] \\
&\quad + \frac{1}{s}|F_\pi(s)|^2\Big[-\frac{\chi_+\chi_-}{2\chi'_+\chi'_-}q_+q_- - \frac{m_\pi^2}{4(\chi'_+)^2}\Big((p_+kp_-k) + 2(p_+Qp_-k) \\
&\quad + 2(p_+kp_-Q)\Big) + \frac{1}{\chi'_+}\Big(\chi_+p_-q_+ + \chi_-p_+q_+ + 2(p_-Qp_+q_+)\Big) + (q_+ \leftrightarrow q_-)\Big] \\
&\quad + \frac{1}{s}\mathrm{Re}\,(F_\pi(s)F_\pi^*(s_1))\Big\{(p_+Qp_-k)\frac{q_+}{\chi'_+}\left(\frac{p_+}{\chi_+} - \frac{p_-}{\chi_-}\right) \\
&\quad + 2(p_+Q - p_-Q) - \frac{1}{\chi'_+}\left((p_+Qq_+k) - (p_-Qq_+k)\right) \\
&\quad + \frac{2Qk\;Qp_+\;p_-q_+}{\chi'_+\chi_-} - \frac{2Qp_-\;Qk\;p_+q_+}{\chi'_+\chi_+} - \frac{Q^2\;\chi_+\;q_+p_-}{\chi'_+\chi_-} \\
&\quad + \frac{Q^2\;p_+q_-}{\chi'_+} + \frac{Q^2\;\chi_-\;p_+q_+}{\chi'_+\chi_+} - \frac{Q^2\;p_-q_-}{\chi'_+} - (q_+ \leftrightarrow q_-)\Big\}, \\
k_\pm &= k - p_\pm\,\frac{\chi_\mp}{p_+p_-}\,, \qquad Q = \frac{1}{2}(q_+ - q_-)\,, \\
t &= -2p_-q_-, \quad t_1 = -2p_+q_+, \quad u = -2p_-q_+, \quad u_1 = -2p_+q_-, \\
\mathrm{d}\Gamma &= \frac{\mathrm{d}^3q_+\mathrm{d}^3q_-\mathrm{d}^3k}{q_-^0q_+^0k^0}\delta^{(4)}(p_- + p_+ - q_- - q_+ - k),
\end{aligned}$$

where we use the notation:

$$(abcd) = \frac{1}{4}\mathrm{Sp}\ \hat{a}\hat{b}\hat{c}\hat{d} = (ab)\ (cd) + (ad)\ (bc) - (ac)\ (bd).$$

After algebraic transformations one can get a more compact expression:

$$\mathrm{d}\sigma^{e^+e^-\to\pi^+\pi^-\gamma} = \frac{\alpha^3}{32\pi^2 s}(R_{s_1s_1} + R_{ss} + R_{ss_1})\ \mathrm{d}\Gamma, \tag{2.16}$$

with:

$$\begin{aligned}
R_{s_1s_1} &= |F_\pi(s_1)|^2\Bigg\{A\,\frac{4s}{\chi_-\chi_+} - \frac{8m_e^2}{s_1^2}\left(\frac{t_1u_1}{\chi_-^2} + \frac{tu}{\chi_+^2}\right) + m_\pi^2\Delta_{s_1s_1}\Bigg\},\\
R_{ss} &= |F_\pi(s)|^2\Bigg\{A\,\frac{4s_1}{\chi'_-\chi'_+} - \frac{8m_\pi^2}{s^2}\left(\frac{tu_1}{(\chi'_+)^2} + \frac{t_1u}{(\chi'_-)^2}\right) + m_\pi^2\Delta_{ss}\Bigg\},\\
R_{ss_1} &= \mathrm{Re}\ (F_\pi(s)F_\pi^*(s_1))\Bigg\{4A\left(\frac{u}{\chi_-\chi'_+} + \frac{u_1}{\chi_+\chi'_-} - \frac{t}{\chi_-\chi'_-} - \frac{t_1}{\chi_+\chi'_+}\right) + m_\pi^2\Delta_{ss_1}\Bigg\},\\
A &= \frac{tu + t_1u_1}{ss_1}, \qquad \Delta_{s_1s_1} = -\frac{4}{s_1^2}\,\frac{(t+u)^2 + (t_1+u_1)^2}{\chi_+\chi_-},\\
\Delta_{ss} &= \frac{2m_\pi^2(s-s_1)^2}{s(\chi'_-\chi'_+)^2} + \frac{8}{s^2\chi'_+\chi'_-}(tt_1 + uu_1 - s^2 - ss_1),\\
\Delta_{ss_1} &= \frac{8}{s_1}\left(\frac{t}{\chi_-\chi'_-} + \frac{t_1}{\chi_+\chi'_+} - \frac{u}{\chi_-\chi'_+} - \frac{u_1}{\chi_+\chi'_-}\right)\\
&\quad + \frac{8}{ss_1}\Bigg[\frac{2(t_1-u) + u_1 - t}{\chi'_-} + \frac{2(t-u_1) + u - t_1}{\chi'_+}\\
&\quad + \frac{u_1 + t_1 - s}{2\chi_-}\left(\frac{u}{\chi'_+} - \frac{t}{\chi'_-}\right) + \frac{u + t - s}{2\chi_+}\left(\frac{u_1}{\chi'_-} - \frac{t_1}{\chi'_+}\right)\Bigg].
\end{aligned}$$

In the ultra–relativistic limit ($s \gg m_\pi^2$) one has to drop in the above formula the terms Δ_{ss}, $\Delta_{s_1s_1}$, Δ_{ss_1}.

For numerical estimations it is useful to separate the most singular part of the differential cross-section and to integrate it analytically. We mean the contribution due to hard collinear photon emission by electrons. We suggest the following procedure. Let us define narrow cones surrounding the momenta of the initial particles. Their opening angle is defined by an auxiliary parameter θ_0. The vertex is taken in the interaction point. A photon emitted by the electron or the positron inside the cones ($\widehat{\boldsymbol{k}\boldsymbol{p}_-} < \theta_0$ or $\widehat{\boldsymbol{k}\boldsymbol{p}_+} < \theta_0$) will be called as a collinear one. On the parameter θ_0 one has to impose the restrictions

$$1 \gg \theta_0 \gg \frac{m_e}{\varepsilon}\,. \tag{2.17}$$

Integrating inside the cones we drop all terms proportional to θ_0^2 [9]. After simple calculations one comes to a formula, where a factorization of a *shifted* Born cross-section can be found. So, the process of collinear photon emission is factorized with respect to the hard process of the annihilation into pions. This is the manifestation of the known factorization theorem [10]. Indeed, we obtain the factorization of large logarithm $L = \ln(s/m_e^2)$

with an accompaniment of several non–leading terms. The dependence on the auxiliary parameter θ_0 should cancel in the sum with the contribution of the integration outside the cones. We will keep $\ln(\theta_0^2/4)$ terms and use them below as a compensator.

The *shifted* cross-section $\mathrm{d}\tilde{\sigma}(z_1, z_2)$ reads

$$\mathrm{d}\tilde{\sigma}(z_1, z_2) = \frac{\alpha^2}{4s}\frac{(Y_1^2 - m_\pi^2/\varepsilon^2)^{3/2}}{z_1^2 z_2^2}\;\frac{(1-c^2)\mathrm{d}\Omega|F_\pi(sz_1z_2)|^2}{z_1 + z_2 + (z_2 - z_1)(1 - m_\pi^2/(\varepsilon^2 Y_1^2))^{-1/2} c}\,, \tag{2.18}$$

where z_1 and z_2 are energy fractions of the *almost real* electron and positron after the emission of collinear photons. The energy fractions $Y_{1,2}$ of the final pions can be found from the following kinematical relations:

$$\begin{aligned}
& y_{1,2}^2 = Y_{1,2}^2 - \frac{4m_\pi^2}{s}, \quad z_1 + z_2 = Y_1 + Y_2, \qquad z_1 - z_2 = y_1 c_- + y_2 c_+, \\
& y_1\sqrt{1 - c_-^2} = y_2\sqrt{1 - c_+^2}, \quad c_- \equiv c, \quad Y_{1,2} = \frac{q^0_{-,+}}{\varepsilon}, \quad c_+ = \cos\widehat{\boldsymbol{p}_-\boldsymbol{q}}_+\,, \\
& Y_1 = -\frac{4m_\pi^2}{s}\,\frac{(z_1 - z_2)c}{2z_1z_2 + [4z_1^2z_2^2 - 4(m_\pi^2/s)((z_1+z_2)^2 - (z_1-z_2)^2c^2)]^{1/2}} \\
& \qquad + \frac{2z_1z_2}{z_1 + z_2 - c(z_1 - z_2)}\,.
\end{aligned}$$

The leading contributions to cross-section, containing large logarithm L, as may be recognized, combine into the kernel of Altarelli–Parisi–Lipatov evolution equation:

$$\begin{aligned}
\mathrm{d}\sigma &= \int \mathrm{d}z_1\mathrm{d}z_2\mathcal{D}^\gamma(z_1)\mathcal{D}^\gamma(z_2)\mathrm{d}\tilde{\sigma}_0(z_1, z_2), \\
\mathcal{D}^\gamma(z) &= \delta(1-z) + \frac{\alpha}{2\pi}(L-1)P^{(1)}(z) + \left(\frac{\alpha}{2\pi}\right)^2\frac{(L-1)^2}{2!}P^{(2)}(z) + \ldots\,, \qquad (2.19)\\
P^{(1)}(z) &= \lim_{\Delta\to 0}\left(\delta(1-z)(2\ln\Delta + \frac{3}{2}) + \Theta(1-z-\Delta)\frac{1+z^2}{1-z}\right), \\
P^{(2)}(z) &= \int\limits_x^1 \frac{\mathrm{d}t}{t}P^{(1)}(t)P^{(1)}\left(\frac{z}{t}\right).
\end{aligned}$$

This formula is valid in the leading logarithmical approximation. We will modify it by including nonleading contributions and using the smoothed exponentiated representation for structure functions [11]:

$$\begin{aligned}
\mathcal{D}(z,s) &= \mathcal{D}^\gamma(z,s) + \mathcal{D}^{e^+e^-}(z,s), \qquad (2.20)\\
\mathcal{D}^\gamma(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\Big[1 + \frac{3}{8}b + \frac{b^2}{16}\Big(\frac{9}{8} - \frac{\pi^2}{3}\Big)\Big] \\
&\quad - \frac{1}{4}b(1+z) + \frac{1}{32}b^2\Big(4(1+z)\ln\frac{1}{1-z} + \frac{1+3z^2}{1-z}\ln\frac{1}{z} - 5 - z\Big), \\
\mathcal{D}^{e^+e^-}(z,s) &= \frac{1}{2}b\Big(1-z\Big)^{\frac{b}{2}-1}\Big[-\frac{b^2}{288}(2L-15)\Big] \\
&\quad + \Big(\frac{\alpha}{\pi}\Big)^2\Big[\frac{1}{12(1-z)}\Big(1 - z - \frac{2m_e}{\varepsilon}\Big)^{\frac{b}{2}}\Big(\ln\frac{s(1-z)^2}{m_e^2} - \frac{5}{3}\Big)^2
\end{aligned}$$

$$
\times \Big(1+z^2+\frac{b}{6}\Big(\ln\frac{s(1-z)^2}{m_e^2}-\frac{5}{3}\Big)\Big)+\frac{1}{4}L^2\Big(\frac{2}{3}\,\frac{1-z^3}{z}+\frac{1}{2}(1-z)
+(1+z)\ln z\Big)\Big]\Theta(1-z-\frac{2m_e}{\varepsilon}),\qquad b=\frac{2\alpha}{\pi}(L-1).
$$

In comparison with the corresponding formula in Ref. [11] we shifted the terms, arising due to virtual e^+e^- pair production corrections, from $\mathcal{D}^{\gamma}$ into $\mathcal{D}^{e^+e^-}$.

The final expression for the corrected cross-section reads as follows:

$$
\begin{aligned}
\mathrm{d}\sigma \;=\; & \int\limits_{z_{\min}}^{1}\mathrm{d}z_1\int\limits_{z_{\min}}^{1}\mathrm{d}z_2\,\mathcal{D}(z_1,s)\mathcal{D}(z_2,s)\mathrm{d}\tilde{\sigma}(z_1,z_2)\Big[1+\frac{2\alpha}{\pi}\,(k(c,sz_1z_2) \\
& +b(sz_1z_2)+a)\Big]\Theta_{\mathrm{cut}}(z_1,z_2)+\Bigg[\frac{\alpha^3}{2\pi^2s^2}\int\limits_{\substack{k^0>\Delta\varepsilon\\ \widehat{\boldsymbol{kp}}_{\pm}>\theta_0}} R_1\Big|_{m_e^2=0}\mathrm{d}\Gamma\;\Theta_{\mathrm{cut}}^{(5)} \\
& +\frac{\alpha}{\pi}\int\limits_{\Delta\varepsilon/\varepsilon}^{1}\frac{\mathrm{d}x}{x}\Big(1-x+\frac{x^2}{2}\Big)\ln\frac{\theta_0^2}{4}\Big(\mathrm{d}\tilde{\sigma}(1-x,1)\Theta_{\mathrm{cut}}(1-x,1) \\
& +\mathrm{d}\tilde{\sigma}(1,1-x)\Theta_{\mathrm{cut}}(1,1-x)\Big)\Bigg]+\Bigg[\frac{\alpha^3}{2\pi^2s^2}\int\limits_{k^0>\Delta\varepsilon}R_2\mathrm{d}\Gamma\;\Theta_{\mathrm{cut}}^{(5)} \\
& +\frac{\alpha}{\pi}2\ln\frac{\Delta\varepsilon}{\varepsilon}\mathrm{d}\tilde{\sigma}(1,1)\Big(2\ln\frac{1-\beta c}{1+\beta c}+\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\Big)\Bigg] \\
& +\frac{\alpha^3}{2\pi^2s^2}\int R_3\mathrm{d}\Gamma\;\Theta_{\mathrm{cut}}^{(5)},\qquad z_{\min}=\frac{2m_\pi}{2\varepsilon-m_\pi}\,. \qquad (2.21)
\end{aligned}
$$

Quantities a, b and k are defined in Eqs. (2.5, 2.6). Schematically this expression can be written as

$$
\mathrm{d}\sigma=\mathrm{d}\sigma^{(1)}+[\mathrm{d}\sigma^{(2)}+C^{(2)}]+[\mathrm{d}\sigma^{(3)}+C^{(3)}]+\mathrm{d}\sigma^{(4)}, \qquad (2.22)
$$

where quantities $C^{(2,3)}$ denote compensators; and terms $\mathrm{d}\sigma^{(2,3,4)}$ denote integrals of $R_{1,2,3}$, respectively. Starting with Eq. (2.16) we can replace in the above expression quantities R_i in the following way:

$$
R_1\Big|_{m_e^2=0}\rightarrow\frac{s}{8}R_{s_1s_1}\Big|_{m_e^2=0},\qquad R_2\rightarrow\frac{s}{8}(R_{ss}+R_{ss_1}),\qquad R_3\rightarrow 0. \qquad (2.23)
$$

Experimental conditions of the final particle detection are encoded by step functions $\Theta_{\mathrm{cut}}(z_1,z_2)$ (for the two–particle final state kinematics) and $\Theta_{\mathrm{cut}}^{(5)}$ (for the three–particle one). They can be imposed explicitly by introducing the restriction of the following kind:

$$
\Theta_{\mathrm{cut}}=\Theta(Y_1-y_{\mathrm{th}})\Theta(Y_2-y_{\mathrm{th}})\Theta(\sin\theta_+-\sin\Psi_0)\Theta(\sin\theta_--\sin\Psi_0), \qquad (2.24)
$$

where $y_{\mathrm{th}}\varepsilon=\varepsilon_{\mathrm{th}}$ is the energy threshold of the detectors, angle Ψ_0 determines the *dead* cones around beam axes unattainable for detection. Angles $\theta_\pm$ define the polar angles of the pions. More detailed cuts can be implemented in a Monte Carlo program [12], using the formulæ given above.

There is a peculiar feature in the spectrum of hard photons. Namely in the end of the spectrum the differential cross-section is proportional to the factor

$$I(s_1) = \frac{1}{s_1}\left(1 - \frac{4m_\mu^2}{s_1}\right)^{3/2}, \tag{2.25}$$

which defines a certain peak. It comes from the Feynman diagrams describing the emission by the initial particles [13].

3 Kaon pair production near threshold

In the case of $K_L K_S$ meson pair production the differential cross-section in the Born approximation reads

$$\frac{\mathrm{d}\sigma_0(s)}{\mathrm{d}\Omega_L} = \frac{\alpha^2\beta_K^3}{4s}\sin^2\theta\,|F_{LS}(s)|^2. \tag{3.1}$$

Here, as well as in the case of pions production, we suggest that the form factor F_{LS} includes also the vacuum polarization operator of the virtual photon. Quantity $\beta_K = \sqrt{1 - 4m_K^2/s}$ is the K–meson velocity in the centre–of–mass frame, and θ is the angle between the directions of motion of the long living kaon and the initial electron.

The corrected cross-sections has the form:

$$\frac{\mathrm{d}\sigma^{e^+e^-\to K_LK_S}(s)}{\mathrm{d}\Omega_L} = \int\limits_0^\Delta \mathrm{d}x\,\frac{\mathrm{d}\sigma_0^{e^+e^-\to K_LK_S}(s(1-x))}{\mathrm{d}\Omega_L}F(x,s), \tag{3.2}$$

where (see Ref. [11])

$$\begin{aligned}
F(x,s) \;=\; & bx^{b-1}\Big[1 + \frac{3}{4}b + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) - \frac{b^2}{24}\left(\frac{1}{3}L - 2\pi^2 - \frac{37}{4}\right)\Big] - b\left(1 - \frac{x}{2}\right) \\
& + \frac{1}{8}b^2\Big[4(2-x)\ln\frac{1}{x} + \frac{1}{x}(1 + 3(1-x)^2)\ln\frac{1}{1-x} - 6 + x\Big] \\
& + \left(\frac{\alpha}{\pi}\right)^2\Big\{\frac{1}{6x}\left(x - \frac{2m_e}{\varepsilon}\right)^b\Big[(2 - 2x + x^2)\left(\ln\frac{sx^2}{m_e^2} - \frac{5}{3}\right)^2 + \frac{b}{3}\left(\ln\frac{sx^2}{m_e^2} - \frac{5}{3}\right)^3\Big] \\
& + \frac{1}{2}L^2\Big[\frac{2}{3}\,\frac{1-(1-x)^3}{1-x} + (2-x)\ln(1-x) + \frac{x}{2}\Big]\Big\}\Theta(x - \frac{2m_e}{\varepsilon}).
\end{aligned} \tag{3.3}$$

We omitted a small contribution (proportional to $\alpha(m_\phi - 2m_K)/m_\phi$) from photon emission by the final particles. Note that at higher energies this effect is extremely interesting: it may shed light on the neutral kaons polarizability problem.

In the case of K^+K^- mesons production the Coulomb final state interaction is to be taken into account:

$$\frac{\mathrm{d}\sigma_0(s)}{\mathrm{d}\Omega_-} = \frac{\alpha^2\beta_K^3}{4s}\sin^2\theta|F_K(s)|^2\frac{Z}{1 - \exp(-Z)}, \tag{3.4}$$

$$Z = \frac{2\pi\alpha}{v}, \qquad v = 2\sqrt{\frac{s - 4m_K^2}{s}}\left(1 + \frac{s - 4m_K^2}{s}\right)^{-1},$$

where v is the relative velocity of kaons [14]. When $s = m_\phi^2$ we have $v \approx 0.5$. Because we consider the energy range close to ϕ mass one may choose the maximal energy of the soft photon as

$$\omega \le \Delta E = m_\phi - 2m_K \ll m_K, \quad \Delta \equiv \frac{\Delta E}{m_K} \approx \frac{1}{25}. \tag{3.5}$$

If required, more precise formulæ for charge–even and charge–odd parts of cross-section may be obtained from the ones for charged pions production process with the replacement $\beta \to \beta_K$.

4 Conclusions

Thus, we presented differential cross-sections to be integrated in concrete experimental conditions. The formulæ are good as for semi–analytical integration, as well as for the creation of a Monte Carlo generator [12]. The idea of our approach was to separate the contributions due to $2 \to 2$ like processes and $2 \to 3$ like ones. The compensating terms allow us to eliminate the dependence on auxiliary parameters in both contributions separately. In the same approach we had considered in paper [4] the processes of large-angle Bhabha scattering and electron-positron annihilation into muons and photons.

Note that all presented formulæ are valid only for large angle processes. Indeed, in the region of very small angles $\theta \approx m_e/\varepsilon$ of final particles with respect to the beam directions there are contributions of double logarithmic approximation [9]. These small angle regions give the main part of the total cross-section in higher orders. We suppose that this kinematics is rejected by experimental cuts.

Numerical computations were done in the case of pion production. In Fig. 1 we give the values of radiative corrections normalized by the Born differential cross-section. The dashed line represents the correction in the leading logarithmic approximation with the $\mathcal{K}$-factor included (according to the first term of Eq. (2.21)). The solid line shows the resulting total values of the corrections after adding of large-angle photon emission (with the compensators). We used the following set of parameters: the energy threshold for pion registration $\Delta_1 = y_{\rm th} = 0.5$, the detector angular acceptance $10° < \theta_\pm < 170°$, the beam c.m.s. energy $\varepsilon = 0.51$ GeV. The peaks of RC for the forward and backward directions are due to rapid falling of the Born cross-section there.

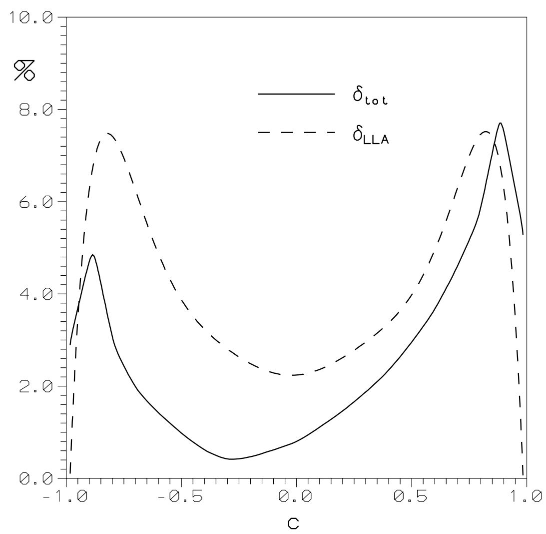

Figure 1: Radiative corrections to the differential cross-section of charged pion production in percent as functions of c.

The second Figure shows the compensation of auxiliary parameters.

The value of $\sigma^{(2)}$ is shown by the solid line. It has to be summed with $C^{(2)}$ which is drawn by middle–dashed line. The value of $\sigma^{(3)}$ is shown by the long–dashed line, and has to be summed with $C^{(3)}$ drawn with the short–dashed line. The parameters for Fig. 2 are: $\Delta_1 = 0.5$, $\Delta = 0.01$, $\theta_0 = 0.01$, other parameters as in Fig. 1. For these numerical illustrations we did not take into account the pion form factor.

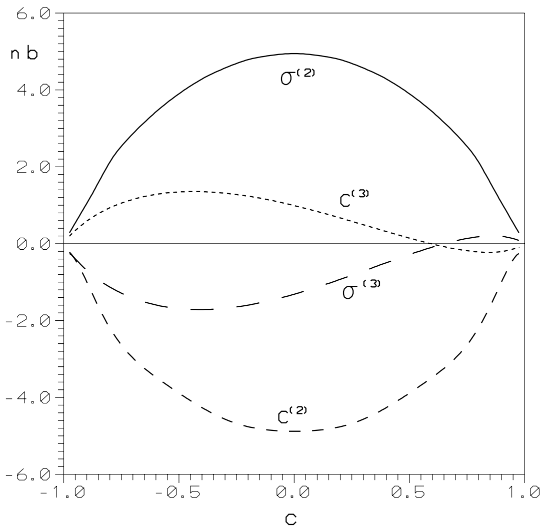

Figure 2: An illustration of the cancellation of the dependence on Δ and θ_0.

The precision of our results is defined by the omitted contributions. As concerns pure QED terms, the corresponding uncertainty is defined by unknown coefficients before the following terms:

$$
\begin{aligned}
&\left(\frac{\alpha}{\pi}\right)^2 L \approx 10^{-4}, \qquad && \left(\frac{\alpha}{\pi}\right)^2 \approx 10^{-5}, \\
&\frac{\alpha}{\pi}\frac{m_e^2}{s}L^2 \approx 10^{-7}, && \frac{\alpha}{\pi}\theta_0^2 \ln(\frac{4}{\theta_0^2}) \approx 10^{-5}, \\
&\frac{\alpha}{\pi}\left(\frac{m_e}{\varepsilon\theta_0}\right)^2 . &&
\end{aligned} \tag{4.1}
$$

We estimate the unknown coefficient[1] and derive the theoretical uncertainty of our calculations for these three processes to be 0.2%. Some additional uncertainty is due to precision of the form factor determination in an experiment. There are also some model–dependent contributions due to different hadronic intermediate states, which are far beyond the scope of this publication.

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[1]For small–angle Bhabha scattering we had performed the complete calculations of next-to-leading logarithmic terms [16]. The corresponding coefficients were found to be about unity.

Appendix

Pion and kaon form factors are supposed to have the form $F_i(s) = (1-\Pi(s))^{-1}\mathcal{F}_i(s)$, where the factors $\mathcal{F}_{\pi,LS,K}(s)$ (unknown theoretically) take into account the strong interactions between hadrons in the final state. We put here the expressions for leptonic and hadronic contributions into vacuum polarization operator $\Pi(s)$:

$$\begin{aligned}
\Pi(s) &= \Pi_l(s) + \Pi_h(s), \qquad\qquad \text{(A.1)}\\
\Pi_l(s) &= \frac{\alpha}{\pi}\Pi_1(s) + \left(\frac{\alpha}{\pi}\right)^2 \Pi_2(s) + \left(\frac{\alpha}{\pi}\right)^3 \Pi_3(s) + \ldots\\
\Pi_h(s) &= \frac{s}{4\pi\alpha}\left[\mathrm{PV}\int\limits_{4m_\pi^2}^{\infty} \frac{\sigma^{e^+e^-\to\mathrm{hadrons}}(s')}{s'-s}\mathrm{d}s' - \mathrm{i}\pi\sigma^{e^+e^-\to\mathrm{hadrons}}(s)\right].
\end{aligned}$$

The first order leptonic contribution is well known [1]:

$$\Pi_1(s) = \frac{1}{3}L - \frac{5}{9} + f(x_\mu) + f(x_\tau) - \mathrm{i}\pi\left[\frac{1}{3} + \phi(x_\mu)\Theta(1-x_\mu) + \phi(x_\tau)\Theta(1-x_\tau)\right], \qquad \text{(A.2)}$$

where

$$\begin{aligned}
f(x) &= \begin{cases} -\frac{5}{9} - \frac{x}{3} + \frac{1}{6}(2+x)\sqrt{1-x}\ln\left(\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\right) & \text{for } x \le 1,\\ -\frac{5}{9} - \frac{x}{3} + \frac{1}{6}(2+x)\sqrt{1-x}\ \mathrm{atan}\left(\frac{1}{\sqrt{x-1}}\right) & \text{for } x > 1,\end{cases}\\
\phi(x) &= \frac{1}{6}(2+x)\sqrt{1-x}, \qquad x_{\mu,\tau} = \frac{4m_{\mu,\tau}^2}{s}.
\end{aligned}$$

In the second order it is enough to take only the logarithmic term from the electron contribution

$$\Pi_2(s) = \frac{1}{4}(L - \mathrm{i}\pi) + \zeta(3) - \frac{5}{24}. \qquad \text{(A.3)}$$

Here we present also a theoretical estimate for the contribution to the vacuum polarization operator due to ϕ meson in the energy region close to the resonance $\sqrt{s} \approx m_\phi \approx 1020$ GeV (see also [5, 17]). In this region one can write the cross-section of e^+e^- annihilation into hadrons as follows:

$$\sigma_h(s) = \frac{12\pi B_{ee}\Gamma_\phi^2}{(s-m_\phi^2)^2 + m_\phi^2\Gamma_\phi^2}, \qquad \text{(A.4)}$$

where B_{ee} is the branching ratio of the decay $\phi \to e^+e^-$, quantity Γ_ϕ is the total width of the meson. For the hadronic part of vacuum polarization connected with ϕ meson we obtain:

$$\Pi_h(s) = \Pi_\phi(s) + \Pi_{h'}(s), \qquad \text{(A.5)}$$

where $\Pi_{h'}(s)$ includes other hadronic contributions [15]. The contribution $\Pi_\phi(s)$ is essential only in the region $m_\phi - n\Gamma_\phi < \sqrt{s} < m_\phi + n\Gamma_\phi$, $n \sim 1$. It has the following form:

$$\Pi_\phi(s) = \frac{3B_{ee}}{\alpha}\,\frac{\frac{\Gamma_\phi}{m_\phi}\left(\frac{s}{m_\phi^2}-1\right)}{\left(\frac{\Gamma_\phi}{m_\phi}\right)^2 + \left(\frac{s}{m_\phi^2}-1\right)^2}, \qquad B_{ee} \approx 3.09\cdot 10^{-4}. \qquad \text{(A.6)}$$

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Erratum

The following changes of the text are in order:

Formula (2.5) should read:

$$\begin{aligned}\mathrm{Re}\,F_\pi^{(1)}(s) &= 1+\frac{\alpha}{\pi}\Bigg\{\left(\ln\frac{m_\pi}{\lambda}-1\right)\left(1-\frac{1+\beta^2}{2\beta}l_\beta\right)+ \\ &\quad +\frac{1+\beta^2}{2\beta}\left[2\xi_2-\frac{1}{4}l_\beta^2+l_\beta\ln\frac{1+\beta}{2\beta}+\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right]\Bigg\}.\end{aligned}$$

The expression $b(s)$ in (2.5) should read:

$$\begin{aligned}b(s) &= -1+\frac{1-\beta}{2\beta}\rho+\frac{2+\beta^2}{\beta}\ln\frac{1+\beta}{2}+ \\ &\quad +\frac{1+\beta^2}{2\beta}\left[\rho+\xi_2+l_\beta\ln\frac{1+\beta}{2\beta^2}+2\mathrm{Li}_2\left(\frac{1-\beta}{1+\beta}\right)\right].\end{aligned}$$

In the formula (2.19) the last line should be:

$$P^{(2)}(z)=\int_z^1\frac{\mathrm{d}t}{t}P^{(1)}(t)P^{(1)}\left(\frac{z}{t}\right).$$

In formula (A.1) the last line should be:

$$\Pi_h(s)=\frac{s}{4\pi^2\alpha}\left[PV\int_{4m_\pi^2}^{\infty}\frac{\sigma^{e^+e^-\to\mathrm{hadrons}}(s')}{s-s'}\mathrm{d}s'-i\pi\sigma^{e^+e^-\to\mathrm{hadrons}}(s)\right].$$

The function $f(x)$ in (A.2) should read

$$f(x)=\begin{cases}-\frac{5}{9}-\frac{x}{3}+\frac{1}{6}(2+x)\sqrt{1-x}\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} & \text{for } x\le 1\,,\\ -\frac{5}{9}-\frac{x}{3}+\frac{1}{3}(2+x)\sqrt{x-1}\arctan\frac{1}{\sqrt{x-1}} & \text{for } x>1\,.\end{cases}$$

The expression (A.6) should read

$$\mathrm{Re}\,\Pi_\phi(s)=\frac{3B_{ee}}{\alpha}\frac{\frac{\Gamma_\phi}{m_\phi}\left(\frac{s}{m_\phi^2}-1\right)}{\left(\frac{\Gamma_\phi}{m_\phi}\right)^2+\left(\frac{s}{m_\phi^2}-1\right)^2},\ B_{ee}\approx 3.09\cdot 10^{-4}.$$